Modeling and Simulation of a Micro-Helicopter in Wind

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ENAE788G Final Project

August 9, 2011

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Contents

Nomenclature

$\mathbb{I}$ moment of inertia tensor
c stabilizer bar linkage scaling constant
l distance from c.g. to tail center of pressure
l_1 distance from c.g. to lower rotor hub
l_2 distance from c.g. to upper rotor hub
L moment component in body frame $\hat{e}_1$ direction
M moment component in body frame $\hat{e}_2$ direction
N moment component in body frame $\hat{e}_3$ direction
p angular velocity component in body frame $\hat{e}_1$ direction
q angular velocity component in body frame $\hat{e}_2$ direction
r angular velocity component in body frame $\hat{e}_3$ direction
$\mathbf{M}$ moment
$\mathbf{D}_t$ drag force on helicopter tail
$\mathbf{D}_b$ drag force on helicopter body
$\mathbf{T}_1$ thrust of bottom rotor
$\mathbf{T}_2$ thrust of top rotor
α roll of TPP
β pitch of TPP
η roll of stabilizer bar spin plane
ζ pitch of stabilzer bar spin plane

ϕ	roll of helicopter
θ	pitch of helicopter
ψ	yaw of helicopter

I Introduction

This paper describes an 8-DOF dynamic model of a coaxial micro-helicopter flying through a steady, uniform wind. The helicopter is modeled as a six degree of freedom rigid body. There are an additional two degrees of freedom contributed to the model by the stabilizer bar located at the top of the helicopter rotor axis. A simulator using this model has been created in MATLAB in order analyze the behavior of the helicopter under various conditions.

The motivation for creating this model is to improve the understanding of the helicopter behavior so that a swarm of helicopters can be effectively used for wind estimation and the implementation of collective control algorithms.

The helicopter is a Blade mCX made by E-flite. It has a rotor diameter of 19cm and a mass of 28g.

Section II describes the coordinates and state variables used in the dynamic model. Section III describes the forces and moments acting on the helicopter during flight and Section IV describes the contribution and behavior of the thrust forces in the model. Section V describes the behavior of the stabilizer bar and its effect on the dynamics of the system. Selected results from simulation are described in Section VI and Section VII gives some insight into the helicopter dynamics through Lagrangian rigid body analysis.

II Coordinates and Frames

The helicopter is modeled as a 3-dimensional rigid body in a right hand Cartesian coordinate system (Figure 1):

$$\begin{aligned} I &= \{O, \hat{e}_x, \hat{e}_y, \hat{e}_z\} \\ B &= \{G, \hat{e}_1, \hat{e}_2, \hat{e}_3\} \end{aligned} \tag{1}$$

The attitude of the helicopter is expressed using a right-hand 3-2-1 Euler rotation sequence. The rotation matrix from frame I to frame B is therefore

$$R_{IB} = \begin{bmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\theta C_\theta & C_\phi C_\theta \end{bmatrix} \tag{2}$$

where ϕ, θ, and ψ are the roll, pitch, and yaw of the helicopter respectively.

R_{IB} is in the SO(3) group of matrices, the opposite transformation $R_{BI} = R_{IB}^T$.

The translational equations of motion are given by Newton's second law

$$\mathbf{F}_{net} = m^I\mathbf{a}_{G/O} = \sum_{i=1}^{n} \bar{F}_i,\ i = 1, 2, 3, ... \tag{3}$$

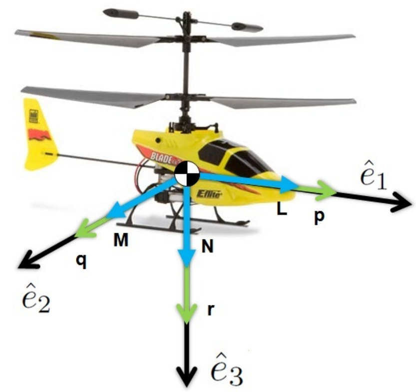


Figure 1: The body frame on the helicopter

where ${}^I\mathbf{a}_{G/O}$ is the inertial acceleration of the center of gravity G of the helicopter.

The rotational equations of motion are expressed in the body frame using Euler's rotational equations of motion[1]

$$\mathbf{M} = \mathbb{I} \cdot^I \dot{\bar{\omega}}^B +^I \bar{\omega}^B \times (\mathbb{I} \cdot^I \bar{\omega}^B) \tag{4}$$

where $\mathbb{I}$ is the moment of inertia tensor for the helicopter and ${}^I\bar{\omega}^B$ is the angular velocity of the helicopter body frame. For simplification, it is assumed that the helicopter has no products of inertia. The inertia tensor is therefore diagonal and defined as

$$\mathbb{I} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \tag{5}$$

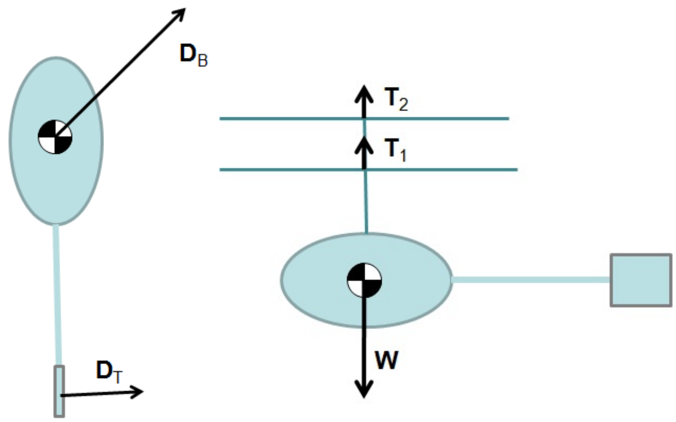
The body frame components of the moment on the helicopter can therefore be expressed as

$$\begin{aligned} M_1 &= I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 \\ M_2 &= I_1\dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1 \\ M_3 &= I_1\dot{\omega}_3 + (I_3 - I_2)\omega_1\omega_2 \end{aligned} \tag{6}$$

where ω_i denotes the body frame component of the angular velocity of the helicopter. In flight dynamics, the body frame components of the angular momentum and angular velocity of the helicopter are commonly redefined as ${}^B\mathbf{M} = [L, M, N]^T$ and $[{}^I\bar{\omega}^B]_B = [p, q, r]^T$. Using this convention, the angular acceleration of the vehicle can be expressed in the body frame as[2]

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \mathbb{I}^{-1} \left(\begin{bmatrix} L \\ M \\ N \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbb{I} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right). \tag{7}$$

The body frame angular velocity components can be transformed into rates of change of the Euler angles using the transformation



(a) Top View (b) Side View

Figure 2: Forces on Helicopter

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & S_\phi \tan\theta & C_\phi \tan\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi \sec\theta & C_\phi \sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{8}$$

III Helicopter Force and Moment Terms

The helicopter has two rotors and each is modeled as a thrust vector in the body frame. The orientation of these thrust vectors is controlled by cyclic inputs sent to each rotor from either the swash plate or stabilizer bar. This is described in detail in Section IV. The drag on the body and tail of the helicopter are the expressed as separate forces. The weight of the helicopter is the also included in the model. The net force on the helicopter can therefore be expressed as

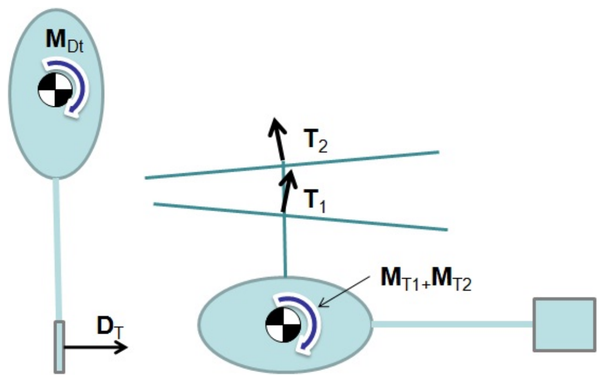
$$\mathbf{F}_{net} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{D}_b + \mathbf{D}_t \tag{9}$$

where $\mathbf{T}_1$ is the thrust vector from the lower rotor and $\mathbf{T}_2$ is the thrust vector from the upper rotor. Vectors $\mathbf{D}_t$ and $\mathbf{D}_b$ are the drag/aerodynamic forces on the tail and body respectively. The forces are illustrated in Figure 2.

The drag terms are calculated using the drag equation

$$D_i = \tfrac{1}{2} c_{D,i} \rho A_i v_i^2 \tag{10}$$

where v_i is the velocity component of the flow with air density ρ projected on a given body axis $\hat{e}_i$. The area A_i is that of the body or tail projected onto the plane perpendicular to $\hat{e}_i$. The drag coefficient $c_{D,i}$ is not easy to calculate. For purposes of simulation it is assumed to be equal to 1.3 for the body and 1.8 for the tail, which is close to the c_D of a flat plate perpendicular to a three dimensional flow.



(a) Top View (b) Side View

Figure 3: Moments on Helicopter

The flow velocity components over the body are calculated by transforming the relative wind vector into body frame components. The drag is then calculated in the body frame and the net force vector due to drag is transformed back into the inertial frame. The relative wind on the tail is different than that on the body because the tail wind-relative velocity is affected by the rotation of the helicopter. If ${}^{I}v_{G/O}$ is the inertial velocity of the body in body frame coordinates, then the inertial velocity of the tail is given by

$$ {}^{I}v_{T/O} = {}^{I} v_{G/O} - lr\hat{e}_2 \tag{11} $$

where l is the distance from the center of pressure of the tail to the helicopter center of mass and r is the $\hat{e}_3$ angular velocity component in the body frame.

The thrust terms act on the helicopter at the hub of each rotor, neither of which are located at the center of mass. The thrust forces therefore exert moments $\mathbf{M}_{T1}$ and $\mathbf{M}_{T2}$ about the center of mass. The drag force on the tail of the helicopter also causes a moment $\mathbf{M}_{Dt}$ to act about the center of mass. It is assumed that the drag force on the body of the helicopter does not contribute to the net moment about the center of mass. In reality, the flapping of the blades slightly contributes to the moment terms[1]; however, it is beyond the scope of this paper and these contributions are assumed to be negligible.

The distance of the hub locations of rotor one and two from the center of mass of the helicopter are denoted as l_1 and 1_2. The rotor hubs are assumed to lie along the $-\hat{e}_3$ axis. Again, the tail is located a distance l along the $-\hat{e}_1$ axis from the center of mass. Therefore, the moment contribution from each rotor and the drag on the tail can be expressed in the body frame as

$$ \mathbf{M} = -l_1\hat{e}_3 \times \mathbf{T}_1 - l_2\hat{e}_3 \times \mathbf{T}_2 + -l\hat{e}_1 \times \mathbf{D}_t \tag{12} $$

The moments on the helicopter are illustrated in Figure 3.

IV Thrust Forces and Tip Path Plane

In hover, with $\mathbf{T} = \mathbf{W}$, the helicopter will maintain a constant position and orientation. A yaw is caused by altering the speed of one or both rotors to create a moment about the $\hat{e}_3$ axis. The forces contributing to this moment are not addressed in this paper. In order to give the helicopter some translational velocity, the rotor plane must tip so that one or both thrust vectors have components along the horizontal inertial plane. In order to do this, a cyclic change in angle of attack is sent to the lower rotor so that the lift of the rotor is highest on the side of the helicopter that will rotate upwards[3]. For example, if a positive roll were desired, the angle of attack (and lift) of the lower rotor blade would be greatest on the left side of the helicopter and lowest on the right side of the helicopter. The alternating lift of the rotor along its path of rotation can be effectively modeled as a tilt of the rotor plane, while maintaining a constant thrust. This is known as the tip path plane (TPP). For the example case, this would correspond to a tip down to the right of the lower rotor plane.

The orientation of the TPP in the body coordinate system can be described using angles α and β. The orientation of the the thrust vector in the body frame is given by the unit vector $\mathbf{n}_{Ti}$[1]

$$\mathbf{n}_{Ti} = \begin{bmatrix} -\cos(\alpha_i)\sin(\beta_i) \\ \sin(\alpha_i) \\ -\cos(\alpha_i)\cos(\beta_i) \end{bmatrix} \tag{13}$$

V Stabilizer Bar

The stabilizer bar on the helicopter gives a cyclic input to the upper rotor when the plane of the spinning stabilizer bar is not parallel to the plane of the upper rotor. The stabilizer bar is linked to the upper rotor through a linkage. This linkage acts to keep the stabilizer bar spinning at the same rate as the upper rotor, which is assumed to lie parallel to the $\{\hat{e}_1, \hat{e}_2\}$ plane (the TPP is not the actual spin plane of the rotor). The linkage also acts as the means through which the stabilizer bar sends a cyclic input and, as a result, changes the orientation of the upper TPP. The effect of this is to add stability to the helicopter system.

If for example, the helicopter was hit by a wind gust that caused a roll perturbation and the lower rotor was not given any cyclic input, the only damping on the oscillation of the angle ϕ would come from drag forces in the absence of a stabilizer bar. However, the stabilizer bar contributes to the damping of this oscillation. The bar itself has a mass at each end and therefore has a relatively high moment of inertia. When the helicopter is hit by the gust, the roll angle of the vehicle is perturbed, but the orientation of the stabilizer bar remains close to its orientation before the perturbation. The difference in orientation causes the stabilizer bar to send a cyclic input to the rotor, which effectively alters the rotors TPP and creates a restoring moment.

To model the difference between the orientation of the stabilizer bar and that of the helicopter, the stabilizer bar is given it's own 2-1 Euler rotation sequence with respect to the inertial frame[1]. The yaw/3-rotation is not considered because we are only concerned with the angle of the rotation plane of the stabilizer bar. The roll and pitch angles of the stabilizer bar are defined as η and ζ respectively.

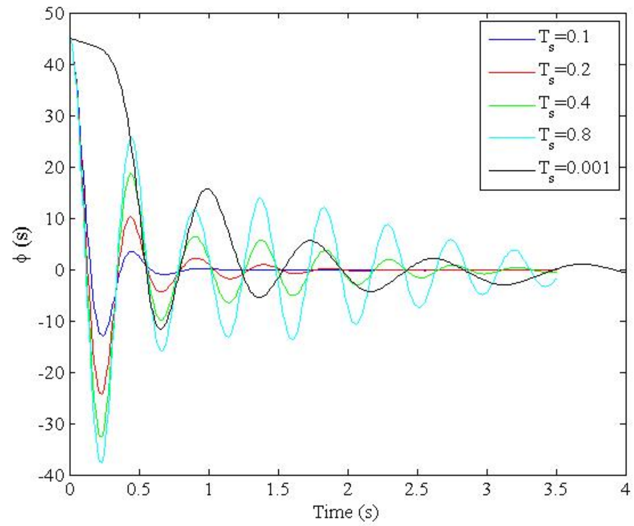


Figure 4: Settling time of roll perturbation at various values of T_s

In the absence of any additional external forces, the stabilizer bar and upper rotor eventually become oriented in a direction parallel to the $\hat{e}_1, \hat{e}_2$ plane due to the transmission of rotor forces through the linkage. The aerodynamic force and linkage interaction is very complex and is beyond the scope of this paper. The dynamics of the stabilizer bar are simplified to a first order system[1]:

$$\begin{aligned} \dot{\eta} &= \tfrac{1}{T_s}(\phi - \eta) \\ \dot{\zeta} &= \tfrac{1}{T_s}(\theta - \zeta) \end{aligned} \tag{14}$$

where T_s is a time constant that is a function of the moment of inertia of the stabilizer bar and the intricacies of the rotor aerodynamics and linkage dynamics. The value of T_s increases with the moment of inertia of the stabilizer bar. Said another way, as the masses at each end of the stabilizer bar increase, the stabilizer bar will tilt more slowly to the horizontal body plane. The ideal value of T_s is not necessarily the time constant corresponding to the highest possible moment of inertia. The optimized settling time of a pitch or roll disturbance is given by a value of T_s that is not too large or too small. To illustrate this, the roll damping of the helicopter given an initial roll of $\phi_0 = \frac{\pi}{4}$ is shown in Figure 4. It can be seen neither the largest nor the smallest time constant plotted gave the best settling time.

The cyclic input from the stabilizer bar and the resulting change in the upper TPP is modeled as

$$\begin{aligned} \alpha_2 &= c(\eta - \phi) \\ \beta_2 &= c(\zeta - \theta) \end{aligned} \tag{15}$$

where α_2 and β_2 are the resulting roll and pitch, respectively, of the upper TPP within the body frame. This transformation was described previously in Equation 13. The constant

c is a scaling factor that is a function of the linkage length between the top rotor and the stabilizer bar.

Identifying T_s parameter would be very difficult. However, we can gain some insight into the effect of the stabilizer bar on the helicopter dynamics by simulating the helicopter under a given flight condition while varying the time constant. Previous work has been done on the identification of this time constant for a coaxial helicopter of similar scale[1]. This study found that $T_s = 0.24s$ for the micro-helicopter studied. Although this cannot be assumed to be the value of T_s for the Blade mCx, it is still likely within the same order of magnitude.

VI Simulation

In order to illustrate the damping effect of the stabilizer bar, the helicopter was simulated given the initial conditions $[\phi_0 = \pi/4, \theta_0 = 0, \psi_0 = \pi/2]$. A uniform wind was directed in the $+\hat{e}_x$ direction. The plots of the rotation angles during the simulation are plotted in Figure 5. It is evident from this plot that every rotational degree of freedom is damped. Both the roll and pitch angles are damped due to the restoring moment of the stabilizer bar and drag forces on the body and the yaw is damped due to the weathercock stability given by the tail fin. Although it is not completely accurate to equate yaw, pitch, and roll with damping of the body axes, for small perturbations, this intuitive visualization is acceptable. In reality it is the rates p,q, and r that are damped by these forces.

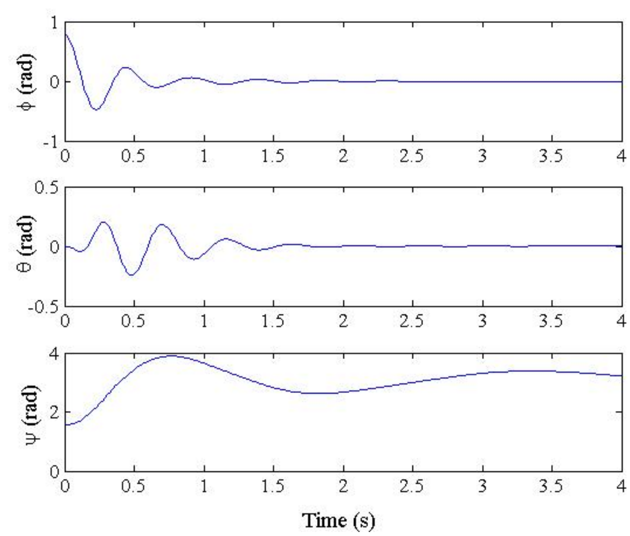


Figure 5: Euler angles vs. time

It is interesting to note the behavior of the angle θ as the helicopter reaches rotational equilibrium. Its oscillations first increase in amplitude because there is a coupling between roll and pitch due to the forces on the tail of the helicopter and $\theta_0 = 0$. However, the oscillations begin to decrease shortly after inception due to the effects of the stabilizer bar.

The thrust for this simulation ($T_1 + T_2$) was set to a magnitude slightly higher than the weight of the helicopter. This resulted in the helicopter losing altitude as is oscillated and then gradually rising up once the vertical component of the thrust was larger than the weight for a long enough period of time.

VII Insights from Lagrangian Analysis

VII.1 Roll stability about equilibrium condition

We can also use the equations of motion derived from a modified simplification of the helicopter model to analyze the roll stability characteristics due to the stabilizer bar. If the model is simplified to be planar and instead of one mass, the mass of the helicopter is split between the body mass m_b and the rotor mass m_h connected by a massless rod (Figure 6), we can gain insight into the contribution of the stabilizer bar to the behavior of the system.

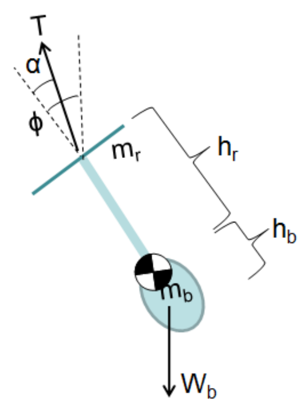


Figure 6: Model for roll stability analysis

The Lagrangian of the proposed system can be expressed as

$$L = \frac{1}{2}(m_h + m_r)(||^I\bar{v}_{G/O}||^2) + \frac{1}{2}I\dot{\phi}^2 - m_b g(y_g - h_b \cos\phi) \tag{16}$$

where I is the moment of inertia of the system about the center of gravity.

The contribution of the mass of the rotor is assumed to be negligible in the potential term of the Lagrangian because $m_r << m_h$ and simplifying the Lagrangian in this way allows for a cleaner, yet still valid, analysis of the system. The reason two masses were included in the model was to initially offset the center of mass of the system from the center of mass of the lower body. Otherwise, the moment of inertia of the system would have been equal to zero. Since we are only concerned with the behavior of ϕ, the relevant force Q for the Lagrangian formulation is the moment about the center of mass due to the thrust. From geometry, one can see that this is equal to $-Th_r \sin\alpha$. Where α is the angle describing the rotation of the TPP in the body frame. The thrust is assumed to have a constant magnitude. The resulting equation of motion for ϕ is

$$\ddot{\phi} = (-h_b m_b g \sin\phi - h_r \sin(\alpha)T)/I_1 \tag{17}$$

which can be rewritten by substituting line one of Equation 15 in for α as

$$\ddot{\phi} = (-h_b m_b g \sin\phi - h_r \sin(\eta - \phi)T)/I_1 = -a\sin\phi - b\sin(\eta - \phi) \tag{18}$$

where a and b are constants greater than zero. The Jacobian for this system is

$$J = \begin{bmatrix} 0 & 1 \\ -a\cos\phi + b\cos(\eta - \phi) & 0 \end{bmatrix} \tag{19}$$

The stable equilibrium solution for this equation is at $\{\dot{\phi}^*, \phi^*\} = \{0, 0\}$. Linearizing about this point, we get

$$A = \begin{bmatrix} 0 & 1 \\ -a + b\cos(\eta) & 0 \end{bmatrix} \tag{20}$$

The eigenvalues of this matrix are

$$\lambda = \pm\sqrt{-a + b\cos\eta} \tag{21}$$

This system will have bounded oscillations only if the eigenvalue is imaginary. Although this eigenvalue has $\mathbb{R}(\lambda) = 0$ and does not prove stability, we know apriori that this is a stable condition and the analysis still give insight into the system behavior. The condition for η is therefore

$$a > b\cos\eta \Rightarrow h_b m_b g > T h_r \cos\eta \Rightarrow \frac{h_b m_b g}{T h_r} > \cos\eta \tag{22}$$

Since $\dot{\eta}$ is defined by a first order system (Equation 14) that is dependent on T_s, the value of η changes in time at a rate dependent upon the moment of inertia and corresponding time constant of the stabilizer bar. This change is the result of the bar changing its pitch η towards the orientation of the $\{\hat{e}_1, \hat{e}_2\}$ plane of the body frame. In other words, during a half sweep of a roll oscillation $\cos\eta$ increases, which decreases the complex magnitude of the eigenvalues. This is has a damping-like effect on the system. In reality, there are additional damping contributions due to energy dissipation in the mechanical linkages and the drag forces on the helicopter. Without this dissipation, the helicopter would continue to oscillate.

This analysis is valid for the pitch behavior as well because the stabilizer bar behaves in the same damping capacity. The only difference would be a slightly different value of I

VII.2 Insight into dissipative forces on the Helicopter

Using the Lagrangian formulation for a three-dimensional rotating rigid body, we can gain insight into the effect of the dissipative forces, namely drag, acting on the helicopter. Consider, at first, the helicopter modeled as a torque-free rigid body. The Lagrangian is defined as

$$L = \frac{I_1}{2}p^2 + \frac{I_2}{2}q^2 + \frac{I_3}{2}r^2 - V \tag{23}$$

where V is the gravitational potential term of the Lagrangian. I_1, I_2, I_3 are the principal moments of inertia of the system. If the angular rates p, q, and r are converted into Euler rates, it can be seen that ψ is a cyclic variable and the system therefore has a canonical momentum

$$p_\psi = \frac{\partial L}{\partial \dot{\psi}} = I_1(\dot{\phi} S_\theta - \dot{\phi} S_\theta^2) + I_2(\theta C_\phi C_\theta S_\phi + \dot{\psi} C_\theta^2 S_\theta^2) + I_3(-\theta C_\theta C_\phi S_\phi + C_\theta C_\phi \dot{\psi}) \quad (24)$$

On the surface, this equation does not lend much insight into the behavior of the helicopter because the actual system is not torque free. However, the canonical momentum (which is also the angular momentum) of the torque free condition can give us insight into the contribution of the real world dissipative forces to the dynamics of the helicopter at a given pose.

Consider the case where $\theta = 0, \phi = \frac{\pi}{4}, \psi = 0$. Equation 24 reduces to

$$p_\psi = \frac{\partial L}{\partial \dot{\psi}} = \dot{\theta} \frac{1}{2}(I_2 - I_3) + \dot{\psi} I_3 \frac{\sqrt{2}}{2} \quad (25)$$

If $I_3 > I_2$ this expression can be further reduced to

$$p_\psi = -A\dot{\theta} + B\dot{\psi}, \quad A, B > 0 \quad (26)$$

Consider further the case where $\dot{\theta}, \dot{\psi} > 0$. In the absence of any moments on the helicopter, the value of p_ψ will have a value that remains unchanged. At the given pose, the contribution of $\dot{\theta}$ and $\dot{\psi}$ to p_ψ is described by constants A and B. These constants also tell us the relative contribution of the damping of both $\dot{\theta}$ and $\dot{\psi}$ to the dissipation of the angular momentum p_ψ. For example, if one is able to determine the damping of the oscillations of $\dot{\theta}$ and $\dot{\psi}$, then one can calculate the relative effect of each damping ratio on the dissipation of the canonical/angular momentum by comparing the coefficients A and B at a given pose. In this case, at some short time step Δt before the current pose, the value of p_ψ was some value g. Without yaw damping, the value of p_ψ at the current pose would still be equal to g. However, due to the presence of damping, this value is slightly less, $g - \Delta g$, where Δg is scaled by the constants A and B in addition to the angle rates $\dot{\psi}$ and $\dot{\theta}$.

VIII Conclusion

The modeling and simulation of the micro-helicopter showed that it is a very stable platform. For essentially all flight conditions for which the model is valid, the helicopter is stable. There may be flight conditions at extreme rotation rates and poses at which the model cannot regain controllability; however, the aerodynamic forces during these flight conditions are complex and the model does not accurately describe them.

Eventual stability of the helicopter is much less important than controllability. That is why understanding the contribution of the stabilizer bar is important when determining how to optimally control the system. This simulation shows that adjusting the time constant T_s for different flight conditions or missions may improve the responsiveness and controllability of the helicopter. The simulator can act as a tool for determining the trade offs between damping and responsiveness from the stabilizer bar. Additionally, understanding the basic aerodynamic forces and moments on the helicopter is important when flying in wind. The model created addresses these forces and can be used as a tool for further analysis of the helicopter dynamics.

The Lagrangian analysis gives some analytical insight into the helicopter dynamics and provides an alternative method of analyzing the helicopter behavior. Whether this will prove to be useful in improving the controllability of the helicopter is yet to be seen.

I Appendix: Parameters used in Model

```
l = 0.1;  %distance from c.g. to a.c of tail. (m)
h = 0.02; %distance from c.g. to a.c. of body (m)
l1 = 0.03; %distance from c.g. to bottom rotor hub
l2 = 0.06;  %distace from c.g. to upper rotor hub
C_dt= 1.8; %drag coefficient of tail
C_db = 1.3; %dreg coefficient of body
rho = 1.221; %density of air  (kg/m^3)
A_t = 0.02*0.03; %area of tail  (m^2)
ra = 0.045; %major axis of ellipse (m)
rb = 0.02; %semimajor axis of ellipse (m)
m = 0.028;%mass of helicopter
I_z = (m/3)*(ra^2+rb^2); %moment of inertia of ellipsoid about vertical axis
I_y = (m/5)*(ra^2+ra^2); %moment of inertia of ellipsoid about vertical axis
I_x = (m/5)*(ra^2+ra^2); %moment of inertia of ellipsoid about vertical axis
A_bf=2*(pi*rb)^2; %Frontal area of ellipsoid
A_bs=2*(pi*ra*rb); %Side and bottom area of ellipsoid
I = [I_x,0,0;0,I_y,0;0,0,I_z]; %Inertia tensor
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